GE 411

Name: ______________	IESE Ph.D Qualifying Exam Quality and Reliability Reliability Portion	Fall 2009 p. 1 of 2 Total Points: 50

Open Book. Permitted: Books, papers, notes, ordinary scientific calculators. No other electronic devices or Laptops. **GE 411 Course Notes are assumed to be in hand.**

Assume statistical independence unless given information to the contrary. Work on these pages.

1. A mechanical component must meet the following requirement to succeed:

$$X^2Y \geq 100$$

X is Lognormal with $\mu_X = 3$ in, $v_X = 20\%$ (i.e., $\lambda_X = 1.079$, $\zeta_X = 0.198$), and material strength Y is Normal with $\mu_Y = 20$ ksi, $v_Y = 15\%$.

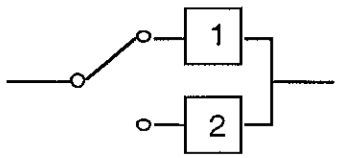
(a) (10 pts) Rewrite the requirement in standard form $g(X, Y) \geq 0$ for success. Find a mean-value FOSM approximate reliability index β without using the AFOSM iterative procedure.

(b) (20 pts) Perform one complete iteration of the design-point method to estimate the AFOSM reliability index β^*, starting at the trial design point ($x^* = 2.5$ in^3, $y^* = 16$ ksi). Find the next trial design point, but do not proceed further. Sketch β^* and the approximate failure boundary in (Z_X, Z_Y) unit normal space.

Note: You may redefine random variables and/or g() to facilitate a simpler solution.

2. (20 pts) Two components in a standby system have Normal wearout lives (no early life or useful life failures). Primary component 1 has a Normal life with $\mu_1 = 1000$ h and $\sigma_1 = 200$ h, and spare component 2 has a Normal life with $\mu_2 = 500$ h and $\sigma_2 = 100$ h. The reliability of the switch is 0.90.

What is the reliability of the standby system for an operating period of 1000 h?



IESE Ph.D. Qualifying Exam (Short 50% version), Fall 2009 Quality Area (1 page) Name:

1. (20 points)

   1.1 Suppose you achieved Cp = 1.67 and your goal is DPM level 2980, how much can your mean deviate and still meet your goal? Assume Target = 0.

   1.2 Suppose you achieved Cpk = 1.67 and CR = 0.5, find the maximum distance between your mean value and the target = 0.

2. A chemical engineer desires to determine the operating conditions that maximizes the yield of a process. An earlier two-level factorial experiment of many considerations indicated that reaction time (x1) and reaction temperature (x2) were the variables that should be optimized. A second-degree model can be fitted using the variables.

   2.1 Fit the data with second-degree model, i.e., y = -1812+*a**x1+17.2*x2-0.05*x1*x1+0.001*x1*x2+*b**x2*x2. (Find *a* and *b*.) (10 points)

   2.2 Find optimal values for the reaction time and reaction temperature. Prove optimality. (20 points)

Responses

x1	x2	Yield
80	170	75.6
80	180	77.0
90	170	78.0
90	180	79.5
85	175	79.9
85	175	80.3
92.07	175	79.8
77.93	175	75.6
85	182.07	78.5
85	167.93	77.0

Z (# of σ)	Pr(z < Zlsl)	DPM		Z (# of σ)	Pr(z > Zusl)	DPM
0.00	0.500000	500000.0		0.00	0.500000	500000.0
-0.25	0.401294	401293.7		0.25	0.401294	401293.7
-0.50	0.308538	308537.5		0,50	0.308538	308537.5
-0.75	0.226627	226627.3		0.75	0.226627	226627.3
-1.00	0.158655	158655.3		1.00	0.158655	158655.3
-1.25	0.105650	105649.8		1.25	0.105650	105649.8
-1.50	0.066807	66807.2		1.50	0.066807	66807.2
-1.75	0.040059	40059.1		1.75	0.040059	40059.1
-2.00	0.022750	22750.1		2.00	0.022750	22750.1
-2.25	0.012224	12224.4		2.25	0.012224	12224.4
-2.50	0.006210	6209.7		2.50	0.006210	6209.7
-2.75	0.002980	2979.8		2.75	0.002980	2979.8
-3.00	0.001350	1350.0		3.00	0.001350	1350.0
-3.25	0.000577	577.1		3.25	0.000577	577.1
-3.50	0.000233	232.7		3.50	0.000233	232.7
-3.75	0.000088	88.4		3.75	0.000088	88.4
-4.00	0.000032	31.7		4.00	0.000032	31.7
-4.25	0.000011	10.7		4.25	0.000011	10.7
-4.50	0.000003	3.4		4.50	0.000003	3.4
-4.75	0.000001	1.02		4.75	0.000001	1.02
-5.00	0.000000	0.29		5.00	0.000000	0.29
-5.25	0.000000	0.08		5.25	0.000000	0.08
-5.50	0.000000	0.019		5.50	0.000000	0.019
-5.75	0.000000	0.004		5.75	0.000000	0.004
-6.00	0.000000	0.001		6.00	0.000000	0.001

GE 424

Ph.D. Qualification Exam in Control Systems

1. Is it possible to design a state feedback control law $u = Kx$ to stabilize the following dynamic system:

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

a) such that the eigenvalues of the closed-loop system are at $\lambda_1 = \lambda_2 = -2$?

b) such that the eigenvalues of the closed-loop system are at $\lambda_1 = -2$ and $\lambda_2 = -3$?

For both cases a) and b): if the system can be stabilized, compute the matrix K.

2. a) Is the following system:

$$\dot{x} = Ax + Bu = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u$$

controllable? λ, b_1, b_2, and b_3 are arbitrary real numbers. Provide a proof to support your answer.

b) Suppose $A = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Find necessary and sufficient conditions on B for controllability of the pair (A, B). Does it matter whether $\mu \neq \lambda$ or $\mu = \lambda$?

IE 410

Qualifying Examination: Stochastic Processes **Fall 2009**

Problem 1

Consider a stationary Markov chain $\ldots, \mathbf{x}_n, \mathbf{x}_{n+1}, \mathbf{x}_{n+2}, \ldots$ with transition probabilities $\{p_{ij}\}$, that is,

$$P(\mathbf{x}_{n+1} = j | \mathbf{x}_n = i) = p_{ij}, \ \forall n,$$

and steady state probabilities $\{q_i\}$.

(a) Show that the reversed sequence $\ldots, \mathbf{x}_n, \mathbf{x}_{n-1}, \mathbf{x}_{n-2}, \ldots$ is also a stationary Markov chain with transition probabilities

$$P(\mathbf{x}_n = j | \mathbf{x}_{n+1} = i) \triangleq p^*_{ij} = \frac{q_j p_{ji}}{q_i}.$$

(b) Show that the Markov chain in part (a) has steady state probabilities $\{q_i\}$.

(c) A Markov chain is said to be time reversible if $p^*_{ij} = p_{ij}$ for all i, j. Show that a necessary condition for time reversibility is that

$$p_{ij} p_{jk} p_{ki} = p_{ik} p_{kj} p_{ji}, \quad \forall i, j, k.$$

Problem 2

Time to failure of a part is distributed exponentially with parameter $\lambda = \frac{1}{2}$ per month, i.e.,

$$f(t) = \lambda e^{-\lambda t}.$$

(a) The part was purchased 2 months ago and was deployed immediately. What is the probability that the part will work for at least one more month?

(b) Can you answer the question in part (a) (i.e, what is the probability that the part will work for at least one more month), if you did not know when it was purchased?

(c) Assume two spare parts are available. Find the probability of surviving six months. (Hint: consider the availability of the spare parts for finding your answer.)

$P(X < 6)$

$(1 - e^{-}$

$P(X < 6)$

$e^{-3} + (1 - e^{-3})(e^{-3})$

$e^{-3}(1 +$

PhD Qualifying Exam: Linear Programming IE 411

Problem 1: Let a_i, $i = 1, 2, 3, 4, 5$ be the vectors given by:

$$a_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad a_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad a_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad a_4 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad a_5 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(a) Consider the set $C \subseteq \mathbb{R}^3$ given by

$$C = \left\{ \sum_{i=1}^{5} \lambda_i a_i \mid \lambda_1, \ldots, \lambda_5 \geq 0 \right\}.$$

Show that any element of C can be expressed in the form $\sum_{i=1}^{5} \lambda_i a_i$ with $\lambda_i \geq 0$ for all i, with at most 3 of the coefficients λ_i being nonzero. *Hint*: Consider the polyhedron

$$\Lambda = \left\{ (\lambda_1, \ldots, \lambda_5) \mid \sum_{i=1}^{5} \lambda_i a_i = y, \ \lambda_1, \ldots, \lambda_5 \geq 0 \right\}.$$

(b) Let P be the convex hull of the vectors $a_i, i = 1, 2, 3, 4, 5$, i.e.,

$$P = \left\{ \sum_{i=1}^{5} \lambda_i a_i \mid \sum_{i=1}^{5} \lambda_i = 1, \ \lambda_1, \ldots, \lambda_5 \geq 0 \right\}.$$

Show that any element of P can be expressed in the form $\sum_{i=1}^{5} \lambda_i a_i$, where $\sum_{i=1}^{5} \lambda_i = 1$ and $\lambda_i \geq 0$ for all i, with at most 4 of the coefficients λ_i being nonzero.

Solution (a) Let y be a point in C and consider the following polyhedron

$$\Lambda = \left\{ \lambda \in \mathbb{R}^5 \mid \sum_{i=1}^{5} \lambda_i a_i = y, \ \lambda_i \geq 0 \text{ for all } i \right\}.$$

The polyhedron Λ is nonempty (since $y \in \Lambda$) and it is in the standard form. Therefore, there exists a basic feasible solution. At a basic feasible solution, we must have 5 active constraints. Hence, at least $5-3 = 2$ of the coefficients λ_i are zero, implying that at most 3 of them are nonzero.

(b) Let y be a point in P and consider the following polyhedron

$$D = \left\{ \lambda \in \mathbb{R}^5 \mid \sum_{i=1}^{5} \lambda_i a_i = y,\ \sum_{i=1}^{5} \lambda_i = 1,\ \lambda_i \geq 0 \text{ for all } i \right\}.$$

The polyhedron D is nonempty (since $y \in D$) and it is in the standard form. Therefore, there exists a basic feasible solution, at which we must have 5 active constraints. Thus, at least $5 - (3 + 1) = 1$ of the coefficients λ_i are zero, implying that at most 4 of them are nonzero.

Problem 2:

Let a and $a_1, a_2, \ldots, a_m$ be given vectors in $\mathbb{R}^n$. Prove that the following two statements are equivalent

(a) $a'x \leq \max_i a_i'x$ for all $x \in \mathbb{R}^n$ with $x \geq 0$.

(b) There exist scalars $\lambda_1, \ldots, \lambda_m$ such that $\lambda_i \geq 0$ for all i, $\sum_{i=1}^{m} \lambda_i = 1$, and

$$a \leq \sum_{i=1}^{m} \lambda_i a_i.$$

Solution Suppose that statement (a) is true. Consider the following LP (primal) problem:

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{i=1}^{m} 0\lambda_i \\ \text{subject to} & \displaystyle\sum_{i=1}^{m} \lambda_i a_i \geq a \\ & \displaystyle\sum_{i=1}^{m} \lambda_i = 1 \\ & \lambda_i \geq 0 \quad \text{for all } i, \end{array}$$

and its dual

$$\begin{array}{ll} \text{maximize} & a'x + y \\ \text{subject to} & a_i'x + y \leq 0 \quad \text{for } i = 1, \ldots, m \\ & x \geq 0, \end{array}$$

where y is a dual variable for the constraint $\sum_{i=1}^{m} \lambda_i = 1$ of the primal.

Let $\bar{x}$ be a dual feasible point. By the dual constraints $a_i'\bar{x} + y \leq 0$ for all i, we obtain

$$y \leq -a_i'\bar{x} \qquad \text{for all } i,$$

implying

$$y \leq -\max_i a_i'\bar{x}.$$

Therefore, for the dual function value, we have

$$a'\bar{x} + y \leq a'\bar{x} - \max_i a_i'\bar{x} \leq 0,$$

where the last inequality follows by condition (a). Thus, the dual problem is feasible and bounded above; hence its optimal value is finite and, conseqeuntly, it has an optimal solution. By the strong LP dulaity it follows that the primal also has an optimal solution, and the primal and the dual optimal values are equal. In particular, it follows that the primal problem must be feasible. Any primal feasible solution satisfies the conditions in (b).

Suppose that statement (b) holds. Then, for every $x \geq 0$, we have

$$a'x \leq \sum_{i=1}^{m} \lambda_i a_i'x \leq \max_j a_j'x \sum_{i=1}^{m} \lambda_i = \max_j a_j'x.$$

Hence, statement (a) holds.