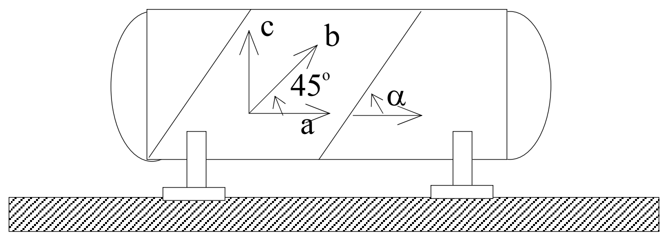
Question 1. The cylindrical portion of a cylindrical air tank is fabricated of steel plate that is welded along a helix that makes an angle of $\alpha=70^o$ with respect to the longitudinal axis of the tank. Before inflation, the length, the inside diameter, and the wall thickness of the cylinder portion of the vessel are 120 in., 48 in., 0.5 in., respectively. Determine the following quantities for the cylindrical portion of the tank assuming an internal pressure of 240 psi.:

a. The axial stresses, σ_a, and hoop stresses, σ_h.
b. The normal stresses and shear stresses on planes parallel and perpendicular to the weld.
c. The maximum in-plane shear stress and the absolute maximum shear stress.
d. The change in length and in diameter of the cylindrical portion after the working internal pressure of 240 psi is applied. $E=30\text{x}10^6$ psi, $\nu=0.3$
e. Assuming that the steel has the yield strength of $30\text{x}10^6$ psi., what is the coefficient of safety using the maximum shear stress theory?



SOLUTION

The axial stresses, σ_a, and hoop stresses, σ_h.

$$\sigma_a = \frac{pr}{2t} = \frac{240\times 24}{2\times 0.5} = 5760\,psi$$

$$\sigma_h = \frac{pr}{t} = \frac{240\times 24}{0.5} = 11520\,psi$$

The normal stresses and shear stresses on planes parallel and perpendicular to the weld.

$$\sigma_c = \sigma_{avg} = \frac{\sigma_a + \sigma_h}{2} = 8640\,psi$$

$$R = \frac{\sigma_a - \sigma_h}{2} = 2880\,psi$$

$$\sigma_n = \sigma_{avg} - R\cos(2\theta)$$

$$= 8640 - 2880\times\cos(40) = 6433\,psi$$

$$\sigma_t = \sigma_{avg} + R\cos(2\theta)$$

$$= 8640 + 2880\times\cos(40) = 10846\,psi$$

$$\tau_{nt} = -R\cos(2\theta)$$

$$= -2880\times\cos(40) = -1851\,psi$$

The maximum in-plane shear stress and the absolute maximum shear stress.

$$\left(\tau_{\max}\right)_{inplane} = R = 2880\,psi$$

$$\left(\tau_{\max}\right)_{absolute} = \frac{\sigma_h}{2} = 5760\,psi$$

The change in length and in diameter of the cylindrical portion after the working internal pressure of 240. psi is applied.

$$\varepsilon_a = \frac{1}{E}\left(\sigma_a - \nu\times\sigma_h\right) = \frac{1}{30\times10^6}\left(5760 - 0.3\times11520\right) = 0.000077\,\frac{in.}{in.}$$

$$\varepsilon_h = \frac{1}{E}\left(\sigma_h - \nu\times\sigma_a\right) = \frac{1}{30\times10^6}\left(11520 - 0.3\times5760\right) = 0.00033\,\frac{in.}{in.}$$

$$\Delta L = L\times\varepsilon_a = 120\times0.000077 = 0.0092in$$

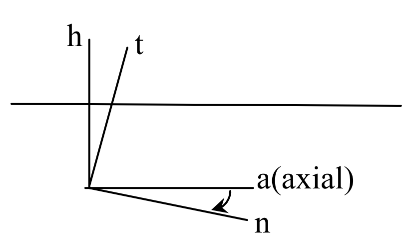
$$P = 2\pi r \Rightarrow \Delta P = 2\pi\Delta r$$

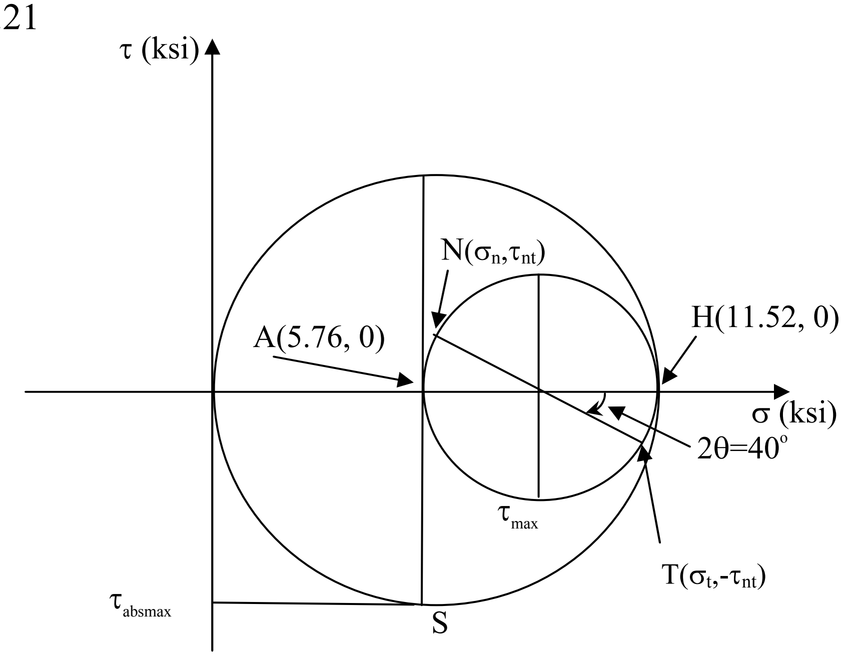
$$\frac{\Delta P}{P} = \frac{2\pi dr}{2\pi r} = \frac{dr}{r} = \varepsilon_h$$

$$\Delta r = r\times\varepsilon_h = 24\times0.00033 = 0.0079in$$

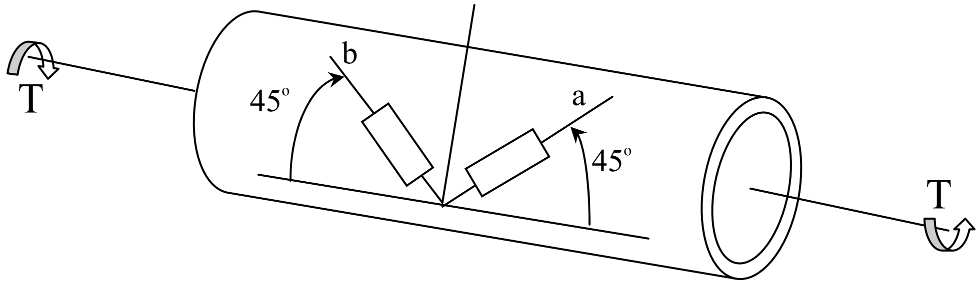
Assuming that the steel has the yield strength of $30\text{x}10^6$ psi., what is the coefficient of safety using the maximum shear stress theory?

$$n = \frac{\sigma_{yield}/2}{\tau_{abs\max}} = \frac{60000/2}{5760} = 5.21$$





Question 2. A Torsion Load cell (to measure torque T) is constructed by mounting two strain gages on a tubular shaft, with gages oriented at 45° to the axis of the tube, as indicated if the following Figure. The gages are wired so that the measurement circuit gives an output $\varepsilon_t = \varepsilon_b - \varepsilon_a$. Determine the relationship between the applied torque T and the measured strain difference, ε_t, if the tube has the following properties: r_o = outer radius; r_i = inner radius; E = modulus of Elasticity; and ν = Poisson's ratio.



Please note that from the orientation of the gages, gage "a" should read compression due to a positive torque, while gage "b" should read tension.

SOLUTION:

$$I_p = \frac{\pi}{2}\left(r_o^4 - r_i^4\right)$$

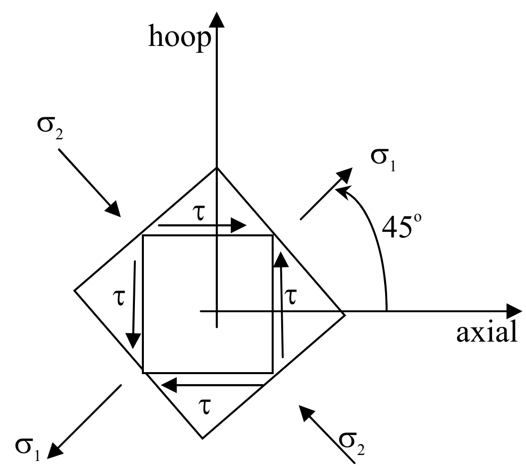
$$G = \frac{E}{2(1+\nu)}$$

$$\varepsilon_b = -\varepsilon_a = \frac{\gamma_{\max}}{2} = \frac{\tau}{2G} = \frac{Tr_o}{2GI_p}$$

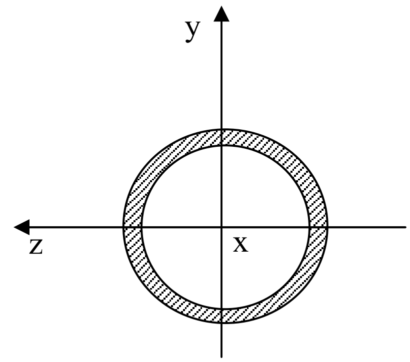
$$\varepsilon_t = \varepsilon_b - \varepsilon_a = 2\varepsilon_b = \frac{\tau}{2G} = \frac{Tr_o}{GI_p}$$

$$T = \left(\frac{GI_p}{r_o}\right)\varepsilon_t = \frac{E}{2(1+\nu)}\left(\frac{\pi}{2r_o}\right)\left(r_o^4 - r_i^4\right)\varepsilon_t$$

$$T = \left[\frac{\pi E\left(r_o^4 - r_i^4\right)}{4(1+\nu)r_o}\right]\varepsilon_t$$



Question 3. A circular tube of outer diameter d_o and inner diameter d_i is made of an elastic –plastic material with yield point σ_y and modulus of elasticity E. Determine expressions for the yield moment M_y, plastic moment M_p, and the shape factor f.



SOLUTION:

$$I = \frac{\pi\left(d_o^4 - d_i^4\right)}{64}$$

$$c = \frac{d_o}{2}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{\sigma_y \pi}{32 d_o}\left(d_o^4 - d_i^4\right)$$

Because the cross-section is symmetric, the neutral axis does not move. Therfore,

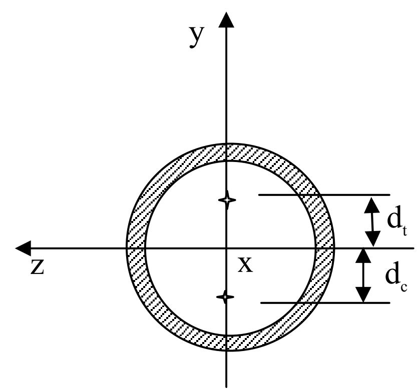
$$d_t = d_c = \frac{\sum \bar{y}_c A_c}{\sum A_c} = \frac{\left(\frac{2d_o}{3\pi}\right)\left(\frac{\pi d_0^2}{8}\right) - \left(\frac{2d_i}{3\pi}\right)\left(\frac{\pi d_i^2}{8}\right)}{\left(\frac{\pi d_0^2}{8}\right) - \left(\frac{\pi d_i^2}{8}\right)} = \frac{2}{3\pi}\left(\frac{d_o^3 - d_i^3}{d_o^2 - d_i^2}\right)$$

$$M_p = \frac{\sigma_y A}{2}\left(d_c + d_t\right)$$

$$= \frac{\sigma_y \pi\left(d_o^2 - d_i^2\right)/4}{2}\left[\frac{2}{3\pi}\left(\frac{d_o^3 - d_i^3}{d_o^2 - d_i^2}\right)\right] \times 2$$

$$= \frac{\sigma_y}{6}\left(d_o^3 - d_i^3\right)$$

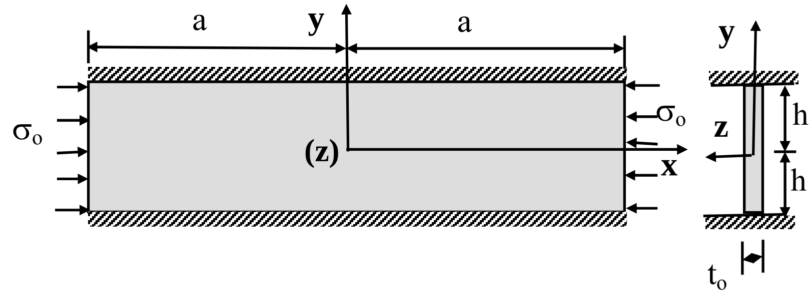
$$f = \frac{M_p}{M_y} = \frac{16 d_o}{3\pi}\left(\frac{d_o^3 - d_i^3}{d_o^4 - d_i^4}\right)$$



Question 4. A long, thin, steel plate of thickness t_o, width 2h, and length 2a is subjected to loads which produce uniform stresses σ_o at the ends as shown in Figure 1. The plate is confined along the edges (y = +h, and y = -h) by two rigid walls. Assuming E and n to be the modulus of elasticity and Poisson's ratio, respectively,

a. Calculate the stress field
b. Calculate the strain field
c. Show that the displacement field is expressed by:

$$u = -\frac{(1-\upsilon^2)}{E}\sigma_o x;$$
$$v = 0;$$
$$w = \frac{\upsilon(1+\upsilon)}{E}\sigma_o z$$



SOLUTION

a. Stress field

$$\sigma_x = -\sigma_o$$
$$\sigma_z = 0$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] = 0 \Rightarrow \sigma_y = \nu(\sigma_x + \sigma_z) = -\nu\sigma_o$$
$$\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

b. Strain field

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] = \frac{1}{E}[-\sigma_o + \nu^2\sigma_0] = -\frac{\sigma_o}{E}(1-\nu^2)$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] = 0$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] = \frac{1}{E}[0 - \nu(-\sigma_o - \nu\sigma_o)] = \frac{\nu\sigma_o}{E}(1+\nu)$$
$$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$$

b. Displacements

$$\varepsilon_x = \frac{\partial u}{\partial x} = -\frac{\sigma_o}{E}(1-\nu^2) \Rightarrow u = -\frac{\sigma_o}{E}(1-\nu^2)x + f(y,z)$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = 0 \Rightarrow v = g(x,z)$$
$$\varepsilon_z = \frac{\partial w}{\partial z} = \frac{\nu\sigma_o}{E}(1+\nu) \Rightarrow w = \frac{\nu\sigma_o}{E}(1+\nu)z + h(x,y)$$

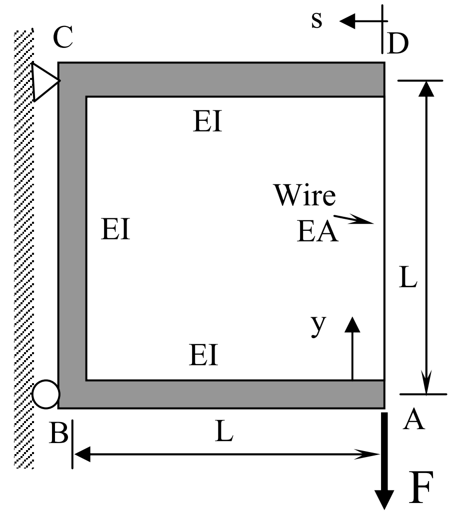
Because of symmetry $u(0,0,0) = v(0,0,0) = w(0,0,0) = 0 \Rightarrow f(y,z) = g(x,z) = h(x,y) = 0$

and

$$u = -\frac{\sigma_o}{E}(1-\nu^2)x$$
$$v = 0$$
$$w = \frac{\nu\sigma_o}{E}(1+\nu)z$$

Question 5. Using the Castigliano's theorem, find the tension T in the wire in terms of F, axial stiffness EA of the wire, and bending stiffness EI of the frame shown. Neglect the effects of axial loading and shear in the members of the frame. T = f(F, EA, EI)

SOLUTION:



A to B→ M1=(T-F)s

B to C→ M2=(T-F)L+Fy

D to C→ M3=Ts

A to D→ N=T

$$U = \frac{1}{2EI} = \left[\int_0^L M_1^2 ds + \int_0^L M_2^2 dy + \int_0^L M_3^2 ds + \right] + \int_0^L \frac{N^2}{2EA} dy$$

$$\frac{\partial U}{\partial N} = 0 \quad \text{(no gap appears at A)}$$

$$0 = \int_0^L (T-F)s^2 ds + \int_0^L [(T-F)L^2 + FLy]dy + \int_0^L Ts^2 ds + \frac{I}{A}\int_0^L Tdy$$

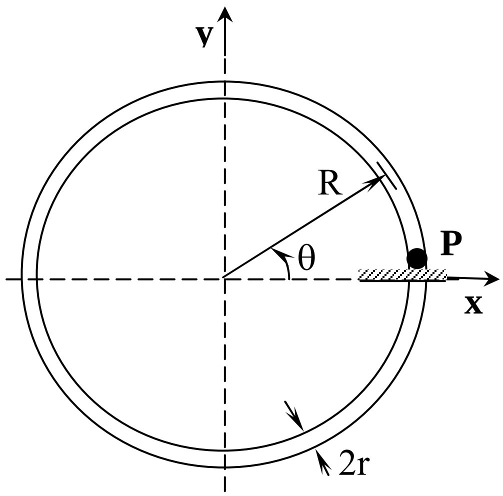
$$0 = L^3 \left[\frac{T}{3} - \frac{F}{3} + T - F + \frac{F}{2} + \frac{T}{3} \right] + \frac{IL}{A} T$$

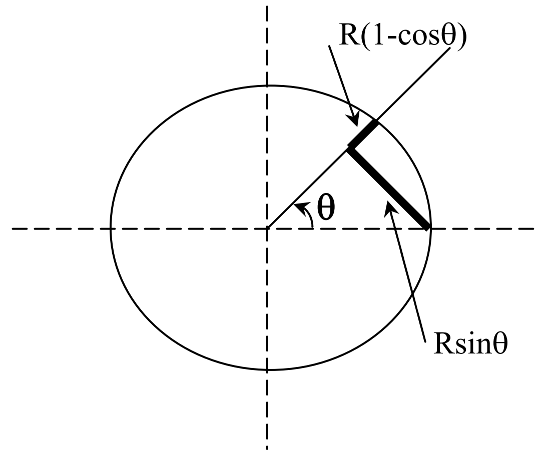
$$0 = \frac{5L^3}{3} T + \frac{IL}{A} T - \frac{5L^3}{6} F$$

or

$$T = \frac{5F/6}{\frac{5}{3} + \frac{I}{AL^2}} = \frac{F}{2 + \frac{6I}{5AL^2}}$$

Question 6. A curved bar with circular cross-section of radius r is fixed at one end as shown. The bar in the form of a split circular ring of radius R is loaded by a force P at the free end applied in a diametric plane perpendicular to the plane of the ring. Using Castigliano's theorem, determine the deflection at the free end.





SOLUTION

$M_b = -PR\sin\theta$ (bending moment)

$M_t = PR(1-\cos\theta)$ (torque)

$$\mathrm{U} = \mathrm{U_b} + U_t = \frac{1}{2EI}\int_0^L M_b^2 dx + \frac{1}{2GJ}\int_0^L M_t^2 dx$$

$$\text{where}: \; J = 2I = \frac{\pi r^4}{2}$$

$$\delta = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{2\pi}(-PR\sin\theta)(-R\sin\theta)Rd\theta + \frac{1}{GJ}\int_0^{2\pi} PR(1-\cos\theta)R(1-\cos\theta)Rd\theta$$

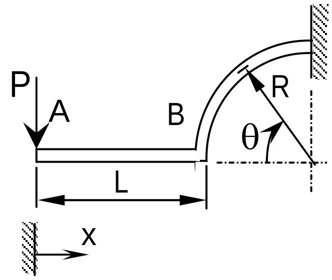
$$\text{Note that}: \; \frac{\partial \mathrm{M_p}}{\partial \mathrm{P}} = -R\sin\theta \quad \text{and} \quad \frac{\partial M_t}{\partial P} = R(1-\cos\theta)$$

$$\text{then} \quad \delta = \frac{PR^3}{EI}\int_0^{2\pi}(\sin\theta)^2 d\theta + \frac{PR^3}{GJ}\int_0^{2\pi}(1-\cos\theta)^2 d\theta$$

After integration

$$\delta = PR^3\pi\left(\frac{1}{EI} + \frac{3}{GJ}\right) \quad \text{where} \quad J = 2I = \frac{\pi r^4}{2}$$

Question 7. For the wire shown in Figure determine the vertical deflection at point A using the Castigliano's theorem and considering bending only. The bending stiffness of the cross-section is EI=constant.



SOLUTION

$$M = -Px \quad \Rightarrow \quad \frac{\partial M}{\partial P} = -x \qquad for \quad 0 \langle x \langle L$$

$$M = P[L + R(1-\cos\theta)] \quad \Rightarrow \quad \frac{\partial M}{\partial P} = L + R(1-\cos\theta) \qquad for \quad 0 \langle \theta \langle \pi/2$$

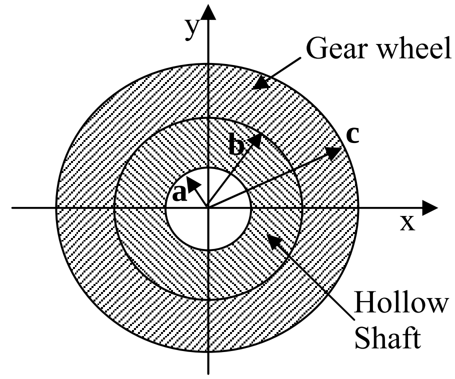
$$\delta = \frac{\partial U}{\partial P} = \frac{1}{EI}\{\int_0^L (-Px)(-x)dx + \int_0^{\pi/2} P[L + R(1-\cos\theta)]^2 R d\theta\}$$

$$\delta = \frac{1}{EI}\{\frac{L^3}{3} + R[(L+R)^2\frac{\pi}{2} - 2R(L+R) + \frac{R^2}{2}\frac{\pi}{2}\}$$

$$\delta = \frac{P}{3EI}\{L^3 + 4.71L^2 R + 3.43LR^2 + 1.07R^3\}$$

Question 8. A gear of inner and outer radii 0.1m and 0.15 m, respectively is shrunk onto a hollow shaft of inner radius 0.05. The length of the gear wheel parallel to the shaft axis is 0.1 m, and the maximum tangential stress induced in the gear wheel by the shrinking process is 0.21 MPa. Assuming a coefficient of friction of 0.2 at the common surface determined the following:

a. The internal pressure developed at the contact surface between the gear and the hollow shaft.
b. The maximum torque that may be transmitted by the gear without slip.



a=0.05 m; b=0.1 m; c=1.5 m

SOLUTION:

Using the theory of thick-walled cylinders, the maximum tangential stress occurs at r=0.1 m in the gear wheel. Then,

$\sigma_{max}=p(a^2+b^2)/(c^2-b^2)$

In which p is the internal pressure developed at the contact surfaces. Upon substitution of given numerical values into the above equation leads to:

$0.21=p[(0.15)^2+(0.1)^2]/[(0.15)^2-(0.1)^2]$

Or,

P=0.081 MPa

This internal pressure at the contact surface controls the maximum torque. The area of contact is:

$2\pi bl=2\pi\ (0.1)(0.1)=0.02\pi$

For a coefficient of friction of 0.2, the torque transmitted is:

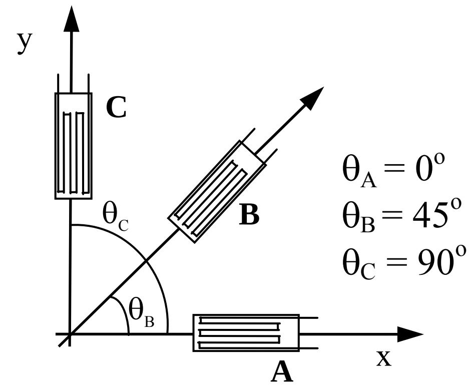
$M_t=0.2(81000 \text{ x } 0.02\pi)(0.1)=1017.36$ N.m

Question 9. A three-element rectangular rosette (see Figure) is applied at a critical location on a steel structure. Under a given loading condition the following readings are recorded:

$$\varepsilon_A = 0\mu\frac{in}{in};\ \varepsilon_B = 500\mu\frac{in}{in};\ \varepsilon_C = 0\mu\frac{in}{in}$$

Assuming the material properties for steel, i.e., modulus of elasticity and Poisson's ratio, to be E = 29 x 10^6 lb/in^2 and ν = 0.29, respectively, determine:

a. The principal strains.
b. The principal stresses.
c. The orientation of the principal axis relative to the strain-gage rosette.



Solution

$$\varepsilon_A = \varepsilon_x cox^2\theta_A + \varepsilon_y \sin^2\theta_A + \gamma_{xy}\sin\theta_A\cos\theta_A$$
$$\varepsilon_B = \varepsilon_x cox^2\theta_B + \varepsilon_y \sin^2\theta_B + \gamma_{xy}\sin\theta_B\cos\theta_B$$
$$\varepsilon_c = \varepsilon_x cox^2\theta_C + \varepsilon_y \sin^2\theta_C + \gamma_{xy}\sin\theta_B\cos\theta_C$$

and you can solve for ε_x, ε_y, and γ_{xy} and then for ε_1 and ε_2,
or if you recognize that it is a state of pure shear, then

$$\varepsilon_{1,2} = \frac{\varepsilon_A - \varepsilon_B}{2} \pm \frac{1}{2}\left[(\varepsilon_A - \varepsilon_C) + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2\right]^{1/2}$$

$$\varepsilon_1 = \frac{1}{2}\sqrt{(2\varepsilon_B)^2} = \varepsilon_B = 500\mu\frac{in}{in}$$

$$\varepsilon_2 = -\frac{1}{2}\sqrt{(2\varepsilon_B)^2} = -\varepsilon_B = -500\mu\frac{in}{in}$$

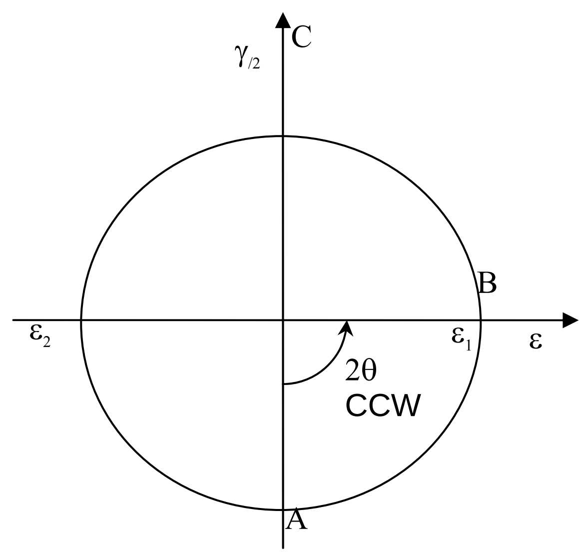
Note that $\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C = -500\mu\frac{in}{in}$

$$2\theta = \tan^{-1}\left(\frac{2\varepsilon_B - \varepsilon_A - \varepsilon_C}{\varepsilon_A - \varepsilon_C}\right) = 90^\circ$$

$\theta = 45^\circ$ CCW from Gage A

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \upsilon\sigma_2) \Rightarrow \sigma_1 = 11{,}240\,psi$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \upsilon\sigma_1) \Rightarrow \sigma_2 = -11{,}240\,psi$$

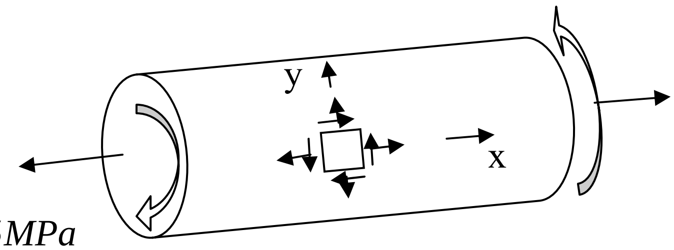


Question 10. A thin-walled tube having internal and external diameters of 0.25 m and 0.26 m is subject to an internal pressure of p_i=2.8 MPa, a twisting moment of 31.36 kN.m, and an axial end thrust (tension) P=45 kN. The ultimate strengths in tension and in compression are 210 MPa and 500 MPa, respectively. Determine if failure has occurred using the following theories of failure.

a) Maximum-shear-stress theory
b) Coulomb-Mohr theory
c) Maximum-principal-stress theory
d) Tresca theory

SOLUTION

Let x be the axial direction,



$$\sigma_x = \frac{pr}{2t} + \frac{P}{\pi(r_o^2 - r_i^2)} =$$

$$= \frac{2800(0.13)}{2(0.005)} + \frac{45}{\pi(0.13^2 + 0.125^2)} = 47.635MPa$$

$$\sigma_y = \frac{pr}{t} = \frac{2.8(0.13)}{0.005} = 72.8MPa$$

$$\tau_{xy} = \frac{31.36(0.13)2}{\pi(0.13^4 - 0.125^4)} = 62.615MPa$$

Principal Stresses are:

$$\sigma_{1,2} = 60.218 \pm [(158.319) + (3920.638)]^{1/2} \Rightarrow \sigma_1 = 124.085MPa \quad and \quad \sigma_2 = -3.649MPa$$

$$a) Maximun - Shear - Stress$$

$$|\sigma_1 - \sigma_2| = \sigma'_{ut}$$

$$127.734 \langle 210 \quad \Rightarrow \quad NO \quad FAILURE$$

$$b) Coulomb - Mohr\ Theory$$

$$\frac{124.085}{210} - \frac{-3.649}{500} \langle\rangle = ? = 1$$

$$0.591 + 0.007 \langle 1 \quad \Rightarrow \quad NO \quad FAILURE$$

$$c) Maximun - \Pr inciple - Stress - Theory$$

$$\sigma_1 ? = \sigma'_{ut} \Rightarrow 124.085 \langle 210 \quad \Rightarrow \quad NO\ \ FAILURE$$

$$\sigma_2 ? = \sigma''_{ut} \Rightarrow 3.649 \langle 500 \quad \Rightarrow \quad NO\ \ FAILURE$$

c) Same as in part a. **No Failure.**